NAG C Library Function Document

nag_zpbtrs (f07hsc)

1 Purpose

nag_zpbtrs (f07hsc) solves a complex Hermitian positive-definite band system of linear equations with multiple right-hand sides, $AX = B$, where A has been factorized by nag_zpbtrf (f07hrc).

2 Specification

```
void nag_zpbtrs (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer kd,
      Integer nrhs, const Complex ab[], Integer pdab, Complex b[], Integer pdb,
      NagError *fail)
```

3 Description

To solve a complex Hermitian positive-definite band system of linear equations $AX = B$, this function must be preceded by a call to nag_zpbtrf (f07hrc) which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If **uplo** = **Nag_Upper**, $A = U^H U$, where U is upper triangular; the solution X is computed by solving $U^H Y = B$ and then $UX = Y$.

If **uplo** = **Nag_Lower**, $A = LL^H$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^H X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether A has been factorized as $U^H U$ or LL^H as follows:

if **uplo** = **Nag_Upper**, $A = U^H U$, where U is upper triangular;

if **uplo** = **Nag_Lower**, $A = LL^H$, where L is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **kd** – Integer *Input*

On entry: k, the number of super-diagonals or sub-diagonals of the matrix A.

Constraint: $\mathbf{kd} \geq 0$.

5: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: $\mathbf{nrhs} \geq 0$.

6: **ab**[dim] – const Complex *Input*

Note: the dimension, dim, of the array **ab** must be at least $\max(1, \mathbf{pdab} \times \mathbf{n})$.

On entry: the Cholesky factor of A, as returned by nag_zpbtrf (f07hrc).

7: **pdab** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **ab**.

Constraint: $\mathbf{pdab} \geq \mathbf{kd} + 1$.

8: **b**[dim] – Complex *Input/Output*

Note: the dimension, dim, of the array **b** must be at least $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pdb} \times \mathbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i,j)th element of the matrix B is stored in $\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i,j)th element of the matrix B is stored in $\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

9: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, $\mathbf{pdb} \geq \max(1, \mathbf{n})$;
if **order** = **Nag_RowMajor**, $\mathbf{pdb} \geq \max(1, \mathbf{nrhs})$.

10: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: $\mathbf{n} \geq 0$.

On entry, **kd** = ⟨*value*⟩.
Constraint: $\mathbf{kd} \geq 0$.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: $\mathbf{nrhs} \geq 0$.

On entry, **pdab** = ⟨*value*⟩.
Constraint: $\mathbf{pdab} > 0$.

On entry, **pdb** = ⟨*value*⟩.
Constraint: $\mathbf{pdb} > 0$.

NE_INT_2

On entry, **pdab** $= \langle value \rangle$, **kd** $= \langle value \rangle$.
Constraint: **pdab** $\geq$ **kd** $+ 1$.

On entry, **pdb** $= \langle value \rangle$, **n** $= \langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** $= \langle value \rangle$, **nrhs** $= \langle value \rangle$.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter $\langle value \rangle$ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

if **uplo** $=$ **Nag_Upper**, $|E| \leq c(k+1)\epsilon|U^H|\,|U|$;

if **uplo** $=$ **Nag_Lower**, $|E| \leq c(k+1)\epsilon|L|\,|L^H|$,

$c(k+1)$ is a modest linear function of $k+1$, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(k+1)\,\text{cond}(A, x)\epsilon$$

where $\text{cond}(A, x) = \||A^{-1}|\,|A|\,|x|\|_\infty / \|x\|_\infty \leq \text{cond}(A) = \||A^{-1}|\,|A|\|_\infty \leq \kappa_\infty(A)$. Note that $\text{cond}(A, x)$ can be much smaller than $\text{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_zpbrfs (f07hvc), and an estimate for $\kappa_\infty(A)$ $(= \kappa_1(A))$ can be obtained by calling nag_zpbcon (f07huc).

8 Further Comments

The total number of real floating-point operations is approximately $16nkr$, assuming $n \gg k$.

This function may be followed by a call to nag_zpbrfs (f07hvc) to refine the solution and return an error estimate.

The real analogue of this function is nag_dpbtrs (f07hec).

9 Example

To solve the system of equations $AX = B$, where

$$A = \begin{pmatrix} 9.39 + 0.00i & 1.08 - 1.73i & 0.00 + 0.00i & 0.00 + 0.00i \\ 1.08 + 1.73i & 1.69 + 0.00i & -0.04 + 0.29i & 0.00 + 0.00i \\ 0.00 + 0.00i & -0.04 - 0.29i & 2.65 + 0.00i & -0.33 + 2.24i \\ 0.00 + 0.00i & 0.00 + 0.00i & -0.33 - 2.24i & 2.17 + 0.00i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -12.42 & + & 68.42i & 54.30 & - & 56.56i \\ -9.93 & + & 0.88i & 18.32 & + & 4.76i \\ -27.30 & - & 0.01i & -4.40 & + & 9.97i \\ 5.31 & + & 23.63i & 9.43 & + & 1.41i \end{pmatrix}.$$

Here A is Hermitian positive-definite, and is treated as a band matrix, which must first be factorized by nag_zpbtrf (f07hrc).

9.1 Program Text

```
/* nag_zpbtrs (f07hsc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer  i, j, k, kd, n, nrhs, pdab, pdb;
  Integer  exit_status=0;
  Nag_UploType uplo_enum;
  NagError fail;
  Nag_OrderType order;

  /* Arrays */
  char    uplo[2];
  Complex  *ab=0, *b=0;

#ifdef NAG_COLUMN_MAJOR
#define AB_UPPER(I,J) ab[(J-1)*pdab + k + I - J - 1]
#define AB_LOWER(I,J) ab[(J-1)*pdab + I - J]
#define B(I,J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define AB_UPPER(I,J) ab[(I-1)*pdab + J - I]
#define AB_LOWER(I,J) ab[(I-1)*pdab + k + J - I - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07hsc Example Program Results\n\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%ld%*[^\n] ", &n, &kd, &nrhs);
  pdab = kd + 1;
#ifdef NAG_COLUMN_MAJOR
  pdb = n;
#else
  pdb = nrhs;
#endif

  /* Allocate memory */
  if ( !(ab = NAG_ALLOC((kd+1) * n, Complex)) ||
       !(b = NAG_ALLOC(n * nrhs, Complex)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A from data file */
  Vscanf(" ' %1s '%*[^\n] ", uplo);
  if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
  else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
  else
    {
      Vprintf("Unrecognised character for Nag_UploType type\n");
      exit_status = -1;
      goto END;
    }
  k = kd + 1;
  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= MIN(i+kd,n); ++j)
            Vscanf(" ( %lf , %lf )", &AB_UPPER(i,j).re, &AB_UPPER(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = MAX(1,i-kd); j <= i; ++j)
            Vscanf(" ( %lf , %lf )", &AB_LOWER(i,j).re, &AB_LOWER(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  /* Read B from data file */
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        Vscanf(" ( %lf , %lf )", &B(i,j).re, &B(i,j).im);
    }
  Vscanf("%*[^\n] ");

  /* Factorize A */
  f07hrc(order, uplo_enum, n, kd, ab, pdab, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07hrc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Compute solution */
  f07hsc(order, uplo_enum, n, kd, nrhs, ab, pdab, b, pdb, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07hsc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Print solution */
  x04dbc(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs, b, pdb,
         Nag_BracketForm, "%7.4f", "Solution(s)", Nag_IntegerLabels,
         0, Nag_IntegerLabels,  0, 80, 0, 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04dbc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  if (ab) NAG_FREE(ab);
  if (b) NAG_FREE(b);
  return exit_status;
}
```

9.2 Program Data

```
f07hsc Example Program Data
  4  1  2                                               :Values of N, KD and NRHS
  'L'                                                       :Value of UPLO
 ( 9.39, 0.00)
 ( 1.08, 1.73) ( 1.69, 0.00)
               (-0.04,-0.29) ( 2.65, 0.00)
                             (-0.33,-2.24) ( 2.17, 0.00)  :End of matrix A
 (-12.42,68.42) (54.30,-56.56)
 ( -9.93, 0.88) (18.32,  4.76)
 (-27.30,-0.01) (-4.40,  9.97)
 (  5.31,23.63) ( 9.43,  1.41)                              :End of matrix B
```

9.3 Program Results

```
f07hsc Example Program Results

 Solution(s)
                    1                   2
 1  (-1.0000, 8.0000)  ( 5.0000,-6.0000)
 2  ( 2.0000,-3.0000)  ( 2.0000, 3.0000)
 3  (-4.0000,-5.0000)  (-8.0000, 4.0000)
 4  ( 7.0000, 6.0000)  (-1.0000,-7.0000)
```